A MULTI-STAGE TURBO-EXPANDER AIR LIQUEFIER

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ABSTRACT. A new type of air liquefier, depending upon the adiabatic expansion of the compressed gas in a multi-stage turbine of low speed, is described. The actual method of liquefaction depends on precooling a suitable fraction of the compressed gas in a heat exchanger, by the other part of the gas cooled by its expansion below the critical point in the turbine, when the former liquefies under its own pressure. The great advantage of such a method is discussed. Preliminary calculations for a small liquefier gives a practical efficiency of about 0.6 kwh. per kgm of liquid air.

INTRODUCTION

In the year 1937, while the author was engaged in developing cryogenic techniques in these laboratories, it occurred to him that a turbine could be used as a refrigerator. Following this, he calculated the details of working of such a turbo-refrigerator as may be used profitably for the liquefaction of air. The author, however, had to leave the construction of the air liquefier unfinished, due to the lack of proper facilities. Recently, owing to the kind facilities offered to the author by the authorities of the Indian Association for the Cultivation of Science, he has been able to take up the work again. On looking through the literature, however, he finds that in the mean time Prof. P. Kapitza has published a paper (Kapitza, 1939) in which he has described the construction of a similar air liquefier in actual action, which has, to a large extent, fulfilled the present author's expectations. The original paper of Prof. Kapitza is not available to the author so that he cannot exactly compare the details of working of Kapitza's machine with his own. It appears, however, from English abstracts of the paper published between 1939-42 that Kapitza's machine is a single stage expansion turbine. The air is compressed to about 6 or 7 atmospheres with the help of a 50 H. P. compressor and after passing through water and air cooler, without any necessity of previous purification, passes into the turbine. This runs at 40,000 r. p. m. and develops about 4 H. P. The pressure of the air drops about 75 per cent in its passage through the turbine and the air entering the turbine, pre-cooled in the last stage to about $-158°C$, emerges at about $-187°C$ and in the liquid state. The weight of the turbine is only about 250 gms. and it deals with about 600 cu. m. of air per hour, producing about 30 kgms. of liquid air per hour, liquefaction beginning within 20 minutes of starting. The efficiency of the liquefier as claimed by Kapitza is about 80 per cent. Hausen, in a subsequent paper (Hausen, 1941) criticises Kapitza's method and says that as regards weight, space occupied and efficiency this machine is in no way superior to the existing Linde and Claude-Heylandt machines. This paper also is not available to the present author so that he cannot discuss the contradictory claims of these two authors, though it would appear from our experience that Hausen's criticism will not most probably be tenable in our case. In any case, one sees clearly the stamp of the typical genius of Prof. Kapitza in the development of this machine.

From what follows, it will be seen that though the principle of Kapitza's new liquefier and that undertaken by the present author are primarily the same, *the details are fundamentally different.* Further, the present author can claim to have *independently* undertaken the project *as early as 1937.* We shall discuss the mreits and demerits of various machines in their proper places but it may even now be mentioned that a Kapitza machine, from what meagre description is available, evidently depends on a *single expansion de Laval impulse turbine*, whereas, the present author uses the *principle of multiple expansion* and various other devices to be mentioned later, which according to thermodynamic principles, is more efficient and at the same time avoids unduly high blade speed, which must be a great drawback of the Kapitza machine. For these reasons author thinks it useful to publish the work undertaken by him, particularly because even after the lapse of 10 years he does not find any attempt by Kapitza to make any further improvement in his machine, which must obviously follow the line suggested by the present author.

From the principles of thermodynamics it is well known that in the production of cold, our aim should be to approximate the ideal reversible isentropic process. Of the two processes at present in use for air liquefaction, the Linde method is essentially irreversible and the maximum performance is about 0.9 kwh. per kgm. of liquid air produced. The Claude-Heylandt method is an attempt towards realising the isentropic method. Unfortunately, due to difficulties of lubrication at low temperatures between the piston and the cylinder in this method, the performance is about the same as in the Linde method. Kapitza has done away with the need of lubrication in his piston-cylinder helium liquefier (Kapitza, 1934) by keeping a very minute clearance between the piston and the cylinder, but this machine can be used only for very small scale generation of cold because of the difficulties in construction of large machines of this type. We know, however, that in a turbine the question of lubrication in the expansion chamber does not arise and a turbine is now a practical success wherever generation of large power is concerned and a large amount of gas is to be dealt with. Thus, in view of the very large demand of liquid air for various laboratories, industrial and other purposes, *e.g.*, scientific investigations at extreme low temperatures, production of oxygen for medical, aviation and submarine breathing purposes, welding, high temperature furnaces, blasting of rocks, etc., and also nitrogen for the manufacture of chemical fertilizers, explosives, etc., it is worthwhile to develop the turbine method of liquefaction of air. In 1926 world's consumption of manufactured oxygen was 80 million cu. m. and of nitrogen 200 million cu. m. In the last two decades the air liquefaction and rectification industry has expanded by leaps and bounds. The by-products of the industry, neon and argon gases are also in great demand.

THERMODYNAMIC PRINCIPLES INVOLVED

Now, let us consider the thermodynamical theory of a heat engine acting as a refrigerator. In an ordinary heat engine a quantity of heat is supplied by the boiler to the working substance at high temperature and a fraction of it is taken up by the condenser at lower temperature as the working substance is made to pass through the complete reversible cycle. The balance of the heat is converted into work done by the engine during the complete cycle. The ideal efficiency is defined by

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} \tag{1}$$

where Q_1 and Q_2 are the amount of heat taken up at the higher temperature T_1 and delivered at the lower temperature T_2 respectively and W is heat equivalent of the work done during the cycle. Also,

$$W = \frac{Q_1 (T_1 - T_2)}{T_1} = \frac{Q_2 (T_1 - T_2)}{T_2} \tag{2}$$

In a refrigerator, on the other hand, we take up an amount of heat Q_2 at the lower temperature T_2 of the condenser, and discharge Q_1 amount of heat at a higher temperature T_1, and in order to do this we have to do work W. Thus, the total energy $Q_2 + W$ supplied $= Q_1$ the amount of heat delivered at the higher temperature and the performance of the refrigerator is

$$\Phi = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2} = \frac{1}{\eta} - 1, \tag{3}$$

also,

$$W = \frac{Q_2 (T_1 - T_2)}{T_2} \tag{4}$$

Thus, the greatest amount of work that is theoretically available in letting a quantity of heat Q_2 pass from a higher temperature to a lower is also the least amount of work that is needed to "pump up" the same quantity of heat through the same range of temperature, the work done depending on temperature as shown. From these considerations it is possible to deduce in a very simple manner many results for a refrigerator by analogy with the existing results for an ordinary heat engine.

It is well known that no engine works in the strictly reversible manner and the "efficiency ratio" is a measure of this deviation from the ideal. The ideal Carnot cycle requires that all the operations upon the working substance should take place in a single vessel so that the entire amount of heat is taken up at a single high temperature. But when the engine is separated into parts, as is done in practice, the last stage of Carnot cycle, *i.e.*, the adiabatic compression from T_2 to T_1, is replaced by a direct heating by the source of heat. In refrigerators, however, this difficulty does not arise. There the performance, as defined by Q_2/W, is increased by taking up heat by the exhaust gas successively at different temperatures and a strict analogy is to be found in the cycle of steam engines with superheating. The total work done in adiabatic expansion is measured by the drop in the total heat or enthalpy, I, during the adiabatic expansion when the gas is admitted at the initial pressure expanded adiabatically and discharged at the final pressure. This measures the amount of cold generated by the machine. The actual work done by the adiabatic expansion is, however, equal to the change of internal energy under adiabatic conditions. The two are denoted by $\int v.dp$ and $\int p.dv$ respectively, where $p.v^\gamma = \text{constant}$, if the working substance is an ideal gas. The net work done upon the working substance in the cycle is given by the area of the cycle (see Fig. 1)

$$abcd = eabcdf - eadf = (I_b - I_c) - (I_a - I_d)$$

$$= \delta I_1 - \delta I_2 = \int_c^b v.dp - \int_d^a v.dp \qquad (5)$$

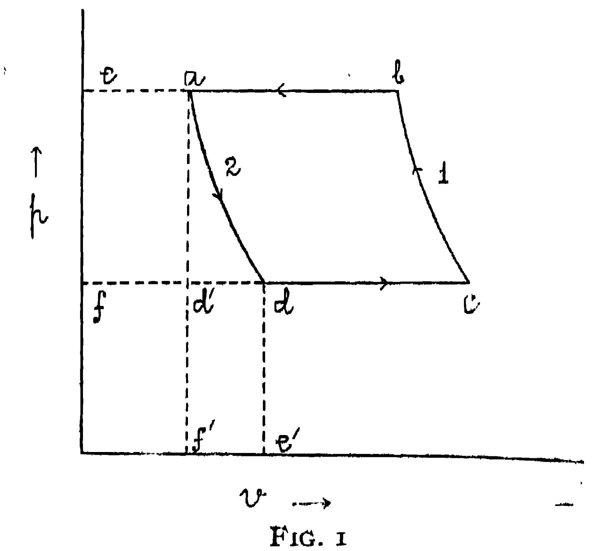


FIG. 1

The cold produced is measured by the change in enthalpy during adiabatic expansion from a to d, *i.e.*, by the area $eadf = \int_d^a v.dp$ ... (6)

The performance

$$\phi = \frac{Q_2}{W} = \frac{eadf}{abcd} = \frac{\delta I_2}{\delta I_1 - \delta I_2} \qquad \dots (7)$$

ADVANTAGES AND DISADVANTAGES OF DIFFERENT TYPES OF EXPANSION ENGINES USED AS REFRIGERATORS

From thermodynamical theories it is evident that for an increased performance of the engine, the heat should be taken up by the expanded gas in stages rather than all being taken up at the lowest temperature. This usually happens in refrigerators by the use of pre-cooling in a counter-current heat exchanger (Ruhemann, 1937). It may also be realised by expanding the gas in multiple stages in two or more expansion machines. Multiple expansion is also closer to the ideal process since the temperature-fall in any particular stage is smaller and the exchange of heat with the engine parts, in consequence, smaller. As is known, this principle of multiple expansion is a necessary condition for the successful working of a turbine from quite different considerations.

To increase the performance of an adiabatic expansion machine, far greater advantage is obtained by reducing the exhaust pressure than increasing the initial pressure, not only because the mechanical difficulties of production and handling of high pressure increases rapidly with increasing pressure but also the works in actual machines corresponding to isothermal compression and adiabatic expansion deviate from the ideal value more at high pressures than at moderately low pressures. Of course, the expansion ratio and the enthalpy change are decreased by using a moderate initial pressure but these may be easily compensated by using a suitable, low exhaust pressure. A further advantage of using lower initial as well as final pressure, is the possibility of using a better heat-exchanger system.

To secure the full benefits of low exhaust pressure, however, the friction between the parts of the expansion machine has to be minimised to the least possible amount so that the expansion may be carried down to the lowest back pressure in the condenser. This is not possible in cylinder and piston engines, not only because (1) the friction of piston against cylinder is quite large, but also (2) the volume becomes excessive. In the turbine these considerations do not arise. Thus, considering (2) first the expansion is carried in stages so that the volume for each expansion is provided in that stage and the turbine blades continuously move on to make space for the expanded gas. On the other hand, the friction occurs only at the shaft bearings which need not be at a low temperature and hence may be properly lubricated. Indeed, the friction is so small that the shaft of a turbine set in motion has been known to revolve for several hours before coming to rest unless a load is put on. Thus, in this way also a turbine from its very construction is eminently suitable as a gas liquefying machine.

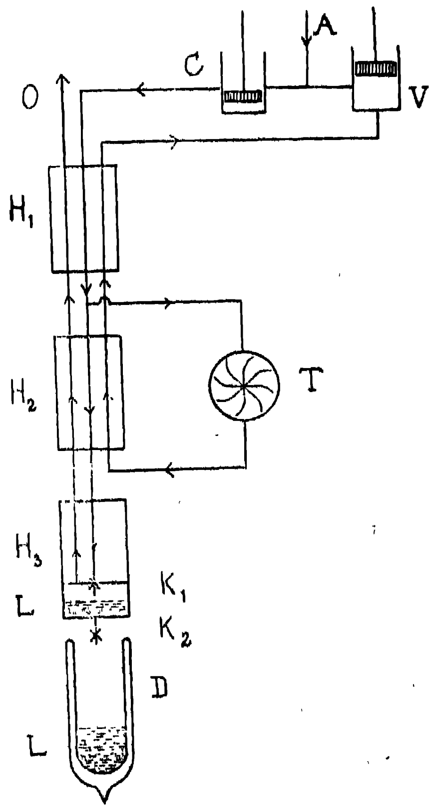
The main disadvantage in a turbine is the leakage over the tips of the blades. But this is negligible for large turbines and particularly if moderate initial pressure is used. Use of moderate pressure is thus advantageous not only for reasons mentioned earlier but also in minimising blade leakage. Hence, use of moderate pressure, like multiple expansion, is an essential condition for the efficient working of a turbine, and this restriction instead of proving to be disadvantageous, really improves the efficiency of the machine over that of high pressure machines, in every way, so much so that no piston and cylinder machine can compare with it. This will be readily seen in the case of "exhaust turbines," so called because they work with the exhaust gas from the piston-cylinder engines and produce nearly as much work.

Another difficulty in a single stage turbine, as used by Kapitza, is the high blade speed, corresponding to about 40,000 r.p.m. But this may be successfully tackled by providing multi-expansion so that the speed is quite moderate, between 2,000-4,000 r.p.m. or even less. At the same time, for reasons mentioned earlier, this multistage expansion should serve to increase the performance as a liquefier. Even this reduced speed may appear very high compared to piston-cylinder engines, but this creates no appreciable mechanical difficulty and is rather useful in reducing the heat leakages by rapid sweeping out of the expanded gases. The turbines are also more efficient than the piston-cylinder engines in this that there are no periodic fluctuations of temperatures in turbine expansion chamber.

The really important loss in a turbine is that due to fluid friction and turbulent motion while the gas passes through the nozzles and the blades and these may be considerable at such high velocities at which the gas passes. Another point to be considered is that turbines can be most profitably used only for really large scale liquefaction of gases. A turbine, handling the same amount of gas as a piston and cylinder machine, has a very much smaller size. This is no doubt very advantageous as regards weight, space occupied by the machine and losses of heat through surface area. But this smallness in size will lead to real difficulty in manufacturing the various parts of the machine when small scale liquefaction is contemplated. For example, if we use a compressor, of capacity 30 cu. m. of air per hour compressed to 50 atmospheres, necessary for the liquefaction of a few litres of liquid air per hour, the size of the cross-section of the turbine nozzle has to be made as small as about 0.01 sq. cm. and the other dimensions of the turbine on a similar scale.

GENERAL PROCESSES AND ARRANGEMENT OF THE METHOD OF LIQUEFACTION

Now, we shall discuss the actual processes of liquefaction with the help of the expansion engine. Since the pressures involved are not high, turbo-compressors capable of delivering large quantities of air may be used with great advantage. The best method of liquefaction appears to us to be cooling down a part of the compressed gas thus obtained, by expansion in a turbine to such a temperature that the other part of the gas, cooled by counter-current heat exchange to this temperature, liquefies under its own pressure. Thus, the final step in liquefaction by Joule-Thomson expansion, as is done by many people but which is an essentially irreversible process and also is not suitable for the low initial pressure used for turbines, is avoided. Liquefaction in the turbine itself as done by Kapitza (1939) would also lead to complications, for example, due to (1) the liquid friction, (2) the high centrifugal force produced by the liquid on the blades and (3) the too low temperatures within the turbine. The amount of cold which has to be generated by the machine to liquefy each kilogramme of air starting from room temperature, say 300°K, is theoretically about 97.6 kilo-calories. The pressure between which we should work is primarily determined by the effective limits of turbine design and also by the lowest temperature we intend to reach in the turbine. This lowest temperature from previous considerations must be above the boiling point of air under the final exhaust pressure but sufficiently below the critical point of air. The diagram of the liquefaction circuit used by us is essentially as shown in Fig. 2, and the various processes of the complete cycle of operations may be very conveniently represented by a temperature-entropy diagram as in Fig. 3.



A—Extra air inlet
V—Vacuum pump
C—Compressor
O—Out let
H_1—1st heat exchanger
H_2—2nd ,, ,,
H_3—3rd ,, ,,
T—Turbo-expander
L—Liquid air
K_1—1st valve
K_2—2nd valve
D—Dewar vessel

FIG. 2

Diagram of liquefier. Arrows mark the passage of air

Supposing that the gas obeys the adiabatic law pv^γ = constant, down to the lowest temperature, the cold developed per unit mass in an "equivalent" single stage expansion, by utilising the kinetic energy of the expanding gas in producing work, is given by

$$\Delta i'.J=\eta.p_1v_1\,\frac{\gamma}{\gamma-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad \ldots \quad (8)$$

where $\Delta i'$ is the adiabatic change of enthalpy in the process and η is the "efficiency ratio" *i.e.*, the ratio of the work done by the actual and the ideal engine. The total change of enthalpy in our arrangement is

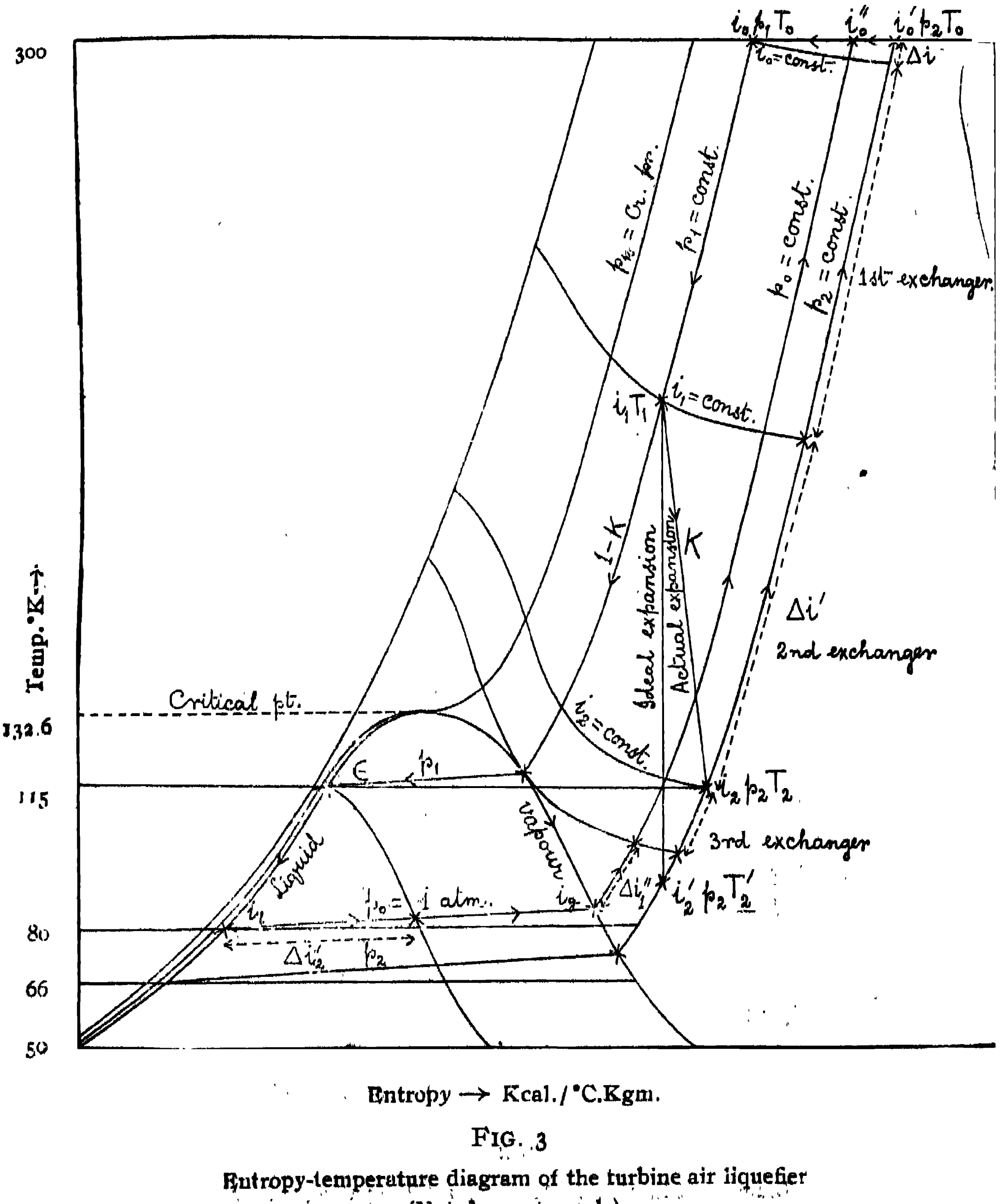


FIG. 3

Entropy-temperature diagram of the turbine air liquefier
(Not drawn to scale)

$Q = \Delta i + K.\Delta i'$, where K is the fraction of air going through the expansion engine, Δi is the isothermal change of enthalpy in compressing the gas at room temperature. (This is, however, only a convenient idealisation).

There is a further amount of change of enthalpy and corresponding cold generated, $(1-K)\Delta i'' = (1-K-\epsilon).\Delta i''_1 + \epsilon.\Delta i''_2$, where ϵ is the fraction of the whole becoming liquid, when the vapour and the liquid phases obtained under high pressure is allowed to pass from pressure p_1 to atmospheric pressure accompanied by enthalpy changes $\Delta i_1''$ and $\Delta i_2''$ respectively, so that really, $Q = \Delta i + K\Delta i' + (1-K)\ \Delta i''$.

If the mixture of vapour and liquid, however, be throttled down to atmospheric pressure through a valve, then of course this last term should be omitted, which will obviously mean a loss of available cold. But at the same time a part of Δi corresponding to change of enthalpy in compressing this part of the gas from atmospheric pressure to pressure p_1 will be made available. This quantity, however, is comparatively small and as we have indicated, we should avoid throttling though we cannot utilize Δi thereby. If we know K, the expanded fraction, and also ϵ, the liquefied fraction, we can easily calculate $(1-K)\Delta i''$ since $\Delta i''$ consists of two parts $\Delta i_1''$, due to change of pressure of the vapour and $\Delta i_2''$ due to that of liquid.

If the gas had obeyed the perfect gas law we might have calculated the amount of cold $K.\Delta i'$, easily. But since this is not so, the quantity is rather difficult to determine. This amount of cold depends in a complicated manner upon the temperature T_1 at which the gas enters the turbine, the initial pressure p_1 and the fraction K, for given values of the efficiency ratio, the final temperature and the pressure. General thermodynamic principles show that the higher the temperature T_1, the larger is the amount of cold, generated by the machine. It is, however, obviously not profitable to make T_1, higher than room temperature. It is to be noted, further, that the higher the final temperature T_2, at the exit of the turbine the higher will T_1 be and less the difference between T_2 and T_0 the initial temperature before entering the heat-exchanger I. Now, the part of the air expanded in the machine must give the whole of its cold to the incoming high pressure air, for which it is necessary that there should exist the least temperature gradient from T_0 to T_2, but this condition may not be fulfilled if $T_0 - T_2$ is too small, so that cold will be lost at the end of the exchanger I. In actual practice this exchanger will be omitted altogether so that $T_0 = T_1$, so that our exchanger-loss consideration will refer to heat-exchanger II. The fraction expanded in the turbine evidently determines the main amount of cold generated and this amount should be only that much as can be efficiently exchanged for a given temperature gradient.

In the ideal case the expansion engine would have taken us to the state i_2', p_2, T_2', from the state i_1, p_1, T_1 but due to losses we reach only i_2, p_2, T_2

and the efficiency ratio is given by

$$\eta=\frac{i_1-i_2}{i_1-i_2'} \qquad \ldots \quad (10)$$

If we pass M kilograms of air through the liquefier of which a fraction K goes through the turbine, the amount of cold generated here is given by $MK\eta\ (i_1-i'_2)$ of which a part is lost in the heat exchangers I and II. There is an additional loss in exchanger III. If the efficiency of the heat exchangers is denoted by ψ then total loss representing the cold taken away by the outgoing gas is given by $MK(1-\psi)(i'_0-i_2)+M(i'_0-i_o)+M(1-\epsilon-K)(1-\psi)(i_0-i_g)$, the first term denoting the loss of cold of the gas passing through the turbine, the last term that of the vapour coming out of the liquefaction chamber and the middle term corresponding to the loss due to the difference of enthalpies at pressures p_1 and p_2 at temperature T_0.

If λ be the amount liquefied by the machine the liquefaction fraction is defined by

$$\epsilon=\frac{\lambda}{M}=\frac{K[\eta(i_1-i_2')-(1-\psi)\ (i_0'-i_2)]-(i_0'-i_0)-(1-\epsilon-K)\ (1-\psi)\ (i_0-i_g)}{L+i_0'-i_g} \quad (11)$$

where L=the latent heat and i_g the enthalpy of the vapour at the boiling point under atmospheric pressure, when the expansion engine alone is producing the cold. There should, however, be another positive added term in the numerator which is due to the change of enthalpy of the liquid-vapour mixture as its pressure changes from p_1 to p_0. This amount is just enough to maintain the vapour-liquid equilibrium while the mixture is passed along the saturation curve so that the amount of gas liquefied remains the same (this is not strictly true for a mixture like air) but may be much more if an additional isentropic expansion is used here. For an accurate calculation of ϵ an elaborate temperature-entropy diagram in which isobars and isenthalps are drawn is necessary and p_1, T_1, K have to be determined empirically. It is obvious that T_2 in our case must be fairly below 132.6°K (critical temperature of air). p_1 not more than about about 40 atmospheres (critical pressure) consistent with good working of the turbine, is then sufficient for liquefaction.

Starting from a given initial state if the value of T_2 is reduced, i_1-i_2' must diminish and in the expression for ϵ the liquefaction fraction, K increases till the product reaches the maximum value giving the maximum production for the liquefier for this value of temperature T_2. Now, if $M(1-K)$ kgm of air be cooled down by heat exchangers at pressure p_1 from T_0 to T_g, the temperature of liquefaction under this pressure, the amount of cooling of the gas is $M(1-K)\psi(i_0-i_g)$. Hence,

$$\epsilon=\frac{\lambda}{M}=\frac{(1-K)\psi(i_0-i_g)}{L+i_0'-i_g} \qquad \ldots \quad (12)$$

From these two equations K can be obtained as a function of temperature T_2.

If the efficiency ratio of the machine is known we can also find out $i_1 - i_2'$ as a function of temperature T_2. The efficiency of heat exchangers are usually about 90%. From the entropy temperature diagram it is not difficult to find out by trial the values of various quantities appearing in the previous equations which give the best performance of the machine. The work supplied to the machine is obtained on the assumption that the compression is isothermal and the gas obeys the Boyles's law during the process (or more accurately from the temperature-entropy diagram).

The consumption of work consists of two parts namely:

(1) compressing the fraction K from pressure p_2 to p_0

(2) and compressing the whole from p_0 to p_1.

If the fraction liquefied be ϵ then it is evident that for the liquefaction of 1 kgm of air $1/\epsilon$ kgm of air must be compressed to the desired pressure. Hence, work consumption,

$$W = \frac{1}{\epsilon}\left[RT(\log p_1/p_0 + K \log p_0/p_2) - K\left(\frac{p_1 v_1 - p_2 v_2}{\gamma - 1}\right)\right] \quad \dots \quad (13)$$

where the negative term takes into account the work recovered from the turbine. Otherwise, the net work consumption may be found out by measuring the area of the temperature-entropy diagram. As K changes the work consumption also changes for given values of p_1 and p_2. The minimum value of work consumption may be calculated from this.

From a rough calculation we see that for an expansion from 10 to .03 atmospheres the suitable initial temperature is about 300°K for a ratio $K = 0.5$. The work consumption is then minimum at $K = 0.5$ and is about 0.6 kwh per kgm of liquid air produced. Actually a transition from 10 atmospheres isobar to 1 atmosphere ought to reach this work consumption at $K = 0.9$ but this is not so due to losses. The final temperature at the exit of the machine is somewhat above 90°K. This rough calculation of the performance compares very favourably with those of other types of machines and with the ideal performance. There is no doubt with further investigations which are in progress in this laboratory, much improvement can be brought about in the machine and there will be no rival to a turbo-liquefier in the field of large scale production of liquid air.

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